JOURNAL

OF THE

WASHINGTON ACADEMY OF SCIENCES

VOL. II, SEPTEMBER 19, 1912 No. 15

MATHEMATICS.—*Account of researches in the algebra of physics.*[1] II. A. MACFARLANE.

The fundamental theorems of analysis generalised for space. Read before the New York Mathematical Society, May 7, 1892. Separately printed. First of all the generalised DeMoivre's Theorem was considered. Let α^a, β^b, γ^c denote three independent spherical angles; then:

$$\alpha^a\beta^b\gamma^c = (\cos a + \sin a \,.\, \alpha^{\pi/2})\ (\cos b + \sin b \,.\, \beta^{\pi/2})$$
$$(\cos c + \sin c \,.\, \gamma^{\pi/2}) = \cos a \cos b \cos c$$
$$+ \cos a \cos b \sin c \,.\, \gamma^{\pi/2} + \cos b \cos c \sin a \,.\, \alpha^{\pi/2}$$
$$+ \cos c \cos a \sin b \,.\, \beta^{\pi/2}$$
$$+ \cos a \sin b \sin c \,.\, \beta^{\pi/2}\ \gamma^{\pi/2} + \cos b \sin c \sin a \,.\, \alpha^{\pi/2}\ \gamma^{\pi/2}$$
$$+ \cos c \sin a \sin b \,.\, \alpha^{\pi/2}\ \beta^{\pi/2} + \sin a \sin b \sin c \,.\, \alpha^{\pi/2}\ \beta^{\pi/2}\ \gamma^{\pi/2}$$

The compound angles are expanded by means of the principle $\beta^{\pi/2}\ \gamma^{\pi/2} = -\cos\beta\gamma - \sin\beta\gamma\ [\beta\gamma]^{\pi/2}$. As the angles are independent of one another, the theorem has its general meaning in the composition of rotations.

The main object of the paper was the demonstration of the generalised Exponential Theorem; from it the other theorems follow as consequences. To prove that

$$e^{b\beta^{\pi/2}}\ e^{c\gamma^{\pi/2}} = e^{b\beta^{\pi/2} + c\gamma^{\pi/2}}$$

[1] Read before the Philosophical Society of Washington, April 20, 1912. See this Journal 2: 331–337. 1912.

$$\text{Since } e^{b\beta^{\pi/2}} = 1 + b\beta^{\pi/2} + \frac{b^2}{2!}\beta^{\pi} + \frac{b^3}{3!}\beta^{3\pi/2} + \cdots$$

$$\text{and } e^{c\gamma^{\pi/2}} = 1 + c\gamma^{\pi/2} + \frac{c^2}{2!}\gamma^{\pi} + \frac{c^3}{3!}\gamma^{3\pi/2} + \cdots$$

$$e^{b\beta^{\pi/2}}\, e^{c\gamma^{\pi/2}} = 1 + c\gamma^{\pi/2} + \frac{c^2}{2!}\gamma^{\pi} + \frac{c^3}{3!}\gamma^{3\pi/2} +$$

$$+ b\beta^{\pi/2} + bc\beta^{\pi/2}\gamma^{\pi/2} + \frac{bc^2}{2!}\beta^{\pi/2}\gamma^{\pi} +$$

$$+ \frac{b^2}{2!}\beta^{\pi} + \frac{b^2c}{2!}\beta^{\pi}\gamma^{\pi/2} +$$

$$+ \frac{b^3}{3!}\beta^{3\pi/2} +$$

$$= 1 + b\beta^{\pi/2} + c\gamma^{\pi/2} + \frac{1}{2!}\left\{b^2\beta^{\pi} + 2bc\beta^{\pi/2}\gamma^{\pi/2} + c^2\gamma^{\pi}\right\}$$

$$+ \frac{1}{3!}\left\{b^3\beta^{3\pi/2} + 3b^2c\beta^{\pi}\gamma^{\pi/2} + 3bc^2\beta^{\pi/2}\gamma^{\pi} + c^3\gamma^{3\pi/2}\right\}$$

$$+ \text{etc.}$$

The question now is: Is the quadratic expression within the brackets the square of $b\beta^{\pi/2} + c\gamma^{\pi/2}$? Hamilton answered in the negative, because it does not conform to the formula

$$(A + B)^2 = (A + B)\ (A + B) = A^2 + AB + BA + B^2.$$

I answer in the affirmative, because it conforms to the formula

$$(A + B)^2 = A^2 + 2AB + B^2.$$

The latter is the square of a succession of vectors A and B, the former is merely the square of the resultant of A and B. Observe also that the above method of multiplying gives only one $bc\beta^{\pi/2}\ \gamma^{\pi/2}$, and that the 2 is introduced to compensate for the factorial 2! placed outside; the multiplication gives no term in $b\ c\ \gamma^{\pi/2}\ \beta^{\pi/2}$ as is given by Hamilton's conjugate square. In a similar manner the terms within the next bracket give $\{b\beta^{\pi/2} + c\gamma^{\pi/2}\}^3$; hence the theorem is proved.

In this way the doctrine of successive vectors and of direct powers was introduced into space-analysis. The truth of the

Binomial Theorem, at least for n being a positive integer, became evident, namely,

$$(iA + iB)^n = (iA)^n + n\ (iA)^{n-1}\ (iB) + \frac{n\,(n-1)}{1\cdot 2}(iA)^{n-1}\ (iB)^2 + \text{etc.}$$

where iA and iB denote two successive logarithmic vectors $a\alpha^{\pi/2}$ and $b\beta^{\pi/2}$; and it was inferred that the same would apply to a sum of successive simple vectors $A + B$ where $A = a\alpha$ and $B = b\beta$.

From the Binomial Theorem, the Multinomial Theorem follows; for example:
$(A + B + C)^2 = A^2 + B^2 + C^2 + 2AB + 2AC + 2BC$; where it is to be noted that the term which occurs is $2AC$ not $2CA$.

It was shown that the sum of successive vectors $A + B - A$ does not reduce to B, but is represented by the three sides of a parallelogram (fig. 3)

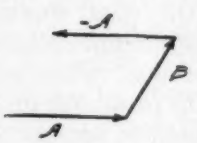

Fig. 3.

On the imaginary of algebra. Proc. A.A.A.S., **41**: 33–55. This paper contains the extension of the prosthaphaerisis theorems of plane trigonometry to spherical trigonometry; also an investigation of the logarithmic circular spiral $e^{\omega\theta^\omega}$, where ω is the constant angle between the radius vector and the tangent. The spiral is equivalent to $\exp\ (b\ \cos\ \omega + b\ \sin\ \omega\ .\ \beta^{\pi/2})$, $= \exp\ b\ \cos\ \omega\ \exp\ b\ \sin\ \omega\ .\ \beta^{\pi/2}$.

The expression, complementary to that for a circular angle, was sought for a hyperbolic angle in space, in the simplest case where the hyperbola is equilateral. The difficulty lay in the circumstance that the logarithm of the hyperbolic β^1 seemed to be β^π, whereas $\beta^{\pi/2}$ seemed to be needed to express the rectangular components. The difficulty was then only partially solved.

Definitions of the trigonometric functions. Read before the Mathematical Congress at Chicago, August 22, 1893. Separately printed. This paper treats of the triangular functions, the circular ratios, the equilateral hyperbolic ratios, the elliptic ratios, and the complex ratios.

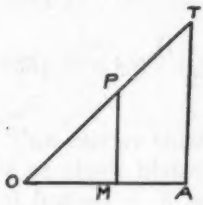

Fig. 4.

Under the first heading are considered two lines OA and OP (fig. 4) with the projections of OP along and perpendicular to OA, namely OM and MP, also certain other projections as AT drawn perpendicular to OA cutting off OT. It was shown that the geometric equation of the first degree

$$OP = OM + MP$$

leads to a geometric equation of the second degree

$$(OA)\ (OP) = (OA)\ (OM) + (OA)\ (MP);$$

that is, the parallelogram (OA) (OP) is equal to the scalar area (OA) (OM) plus the vector area (OA) (MP). This was stated to be the fundamental principle of vector-analysis. The complete product (whose existence is ignored by vector-analysts) is simply the parallelogram formed by the two given lines OA and OP. Its unit, on account of its obliquity partakes partly of the nature of the scalar unit, and partly of the nature of the orthogonal unit. The fundamental principle is expressed by

$$OA)\ (OP) = (OA)\ (OM) + (OA)\ (MP)$$ (fig. 5.)

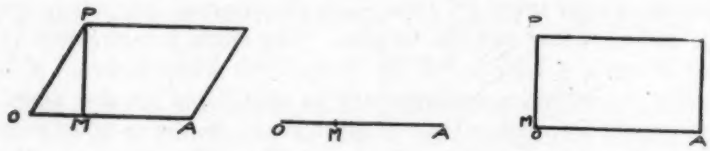

Fig. 5.

In a similar manner, as $OA = OT - AT$, it follows that

$$(OA)\ (OA) = (OA)\ (OT) - (OA)\ (AT)$$

It was pointed out that there is a variety of ways of defining the

circular ratios, differing principally in the line units chosen for the ratio; and that for the other curves these ratios, differing in conception, may cease to have equal numerical values. For example, PM is said to be drawn perpendicular to OA (fig. 6), or parallel to the tangent at A; and cos AOP is variously given by different authors as OM/OA or OM/OP; while sin AOP with still greater ambiguity is defined as

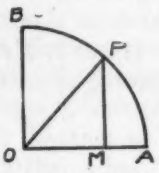

Fig. 6.

PM/OA MP/OA PM/OP MP/OP MP/OB PM/OB.

The simplest ratio is obtained where the two lines of the quotient have the same directed unit; thus OM/OA and MP/OB give pure numerical values, whereas OM/OP and MP/OA involve a difference of direction. In the above paper I defined cos AOP as OM/OA, but made the mistake of defining sin AOP as MP/OA. The simple principle mentioned allows definitions to be given of the circular ratios which apply without change to the more complex curves mentioned.

The principles of elliptic and hyperbolic analysis. Abstract read before the Mathematical Congress at Chicago, August 24, 1893. Separately printed. This paper investigates some of the fundamental principles of trigonometry on the surface of the exsphere, by which is meant the surface of the equilateral hyperboloid. Let AQ (fig. 7) be the positive equilateral hyperbola, A′Q′ the negative, and P″Q″ the conjugate; when the figure revolves about OA, the whole surface traced out by the three curves forms the exsphere. The angle AOP was defined as the ratio of the area of the sector AOP in square units to the area of the triangle AOB in square units; that is $u = A/\frac{a^2}{2}$ that is $2A/a^2$. All the radii from O to the bound-

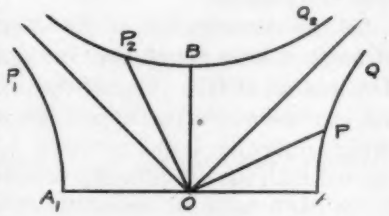

Fig. 7.

ing surface are in a sense unit axes, or unit lines, because the modulus is unity for each.

It was first shown that for a simple hyperbolic angle in a plane passing thru the axis of revolution, the expression is

$$\beta^{ib} \text{ or } \exp ib\beta^{\pi/2} \text{ where } \beta \text{ is a spherical axis.}$$

Let ρ denote any spherical axis; it is expressed quite generally by

$$\rho = \cos\theta \,.\, h + \sin\theta\,(\cos\varphi \,.\, j + \sin\varphi \,.\, k);$$

and if u denote the circular measure, any spherical angle with ρ for axis, that is, in the plane normal to ρ, is expressed by ρ^u or $\exp u\rho^{\pi/2}$.

In a similar manner any axis, as OP, to the double sheet can be expressed as $\rho = \cosh\theta \,.\, h + i \sinh\theta\,(\cos\varphi \,.\, j + \sin\varphi \,.\, k)$; and if u denote an angle of the circular kind, ρ^u or $\exp u\rho^{\pi/2}$ expresses an elliptic angle, that is a sector of the elliptic section made by the plane normal to ρ.

The axis OP'' at right angles to OP, terminated in the single sheet, has the form $i\rho$ and $i\rho = i\{\cosh\theta\,(\cos\varphi \,.\, j + \sin\varphi \,.\, k) - i \sinh\theta \,.\, h\}$. Consequently $i\rho^u = \rho^{iu} = \exp iu\rho^{\pi/2}$ expresses a hyperbolic angle; and the plane normal to $i\rho$ makes a hyperbolic section. The composition of two general exspherical angles was investigated.

In the investigation of the elliptic trigonometry, the simplest definitions were not chosen, the sine being defined with respect to OA instead of OB. One of the main difficulties was the want of an expression for the hyperbolic or elliptic arc, the solution of which difficulty I did not then perceive. The trouble is connected with the old difficulty of the rectification of such arcs.

On the analytical treatment of alternating currents. Proc. of the International Electrical Congress, Chicago, 1893, pp. 24–32. In this paper I pointed out that plane algebra was the proper analytical method for dealing with alternating currents. It was read before Section A, Professor Rowland in the chair. Mr. Steinmetz contributed to the same section an elaborate paper to the same effect, entitled "Complex quantities and their use in electrical engineering." Rowland stated that there was no

doubt about the importance of the subject in the then state of electrical science.

Application of hyperbolic analysis to the discharge of a condenser. Trans. Amer. Inst. Elec. Engineers, 14: 163–174. The investigation of the discharge of a condenser leads to a differential equation, the solution of which depends on the solution of a quadratic equation. The author proceeded on the following theory of the quadratic equation. So far as line algebra is concerned the roots of a quadratic equation with real coefficients are either both real, or else conjugate complexes, the complex roots being scalar in both terms. But for plane algebra the roots of such an equation are either two conjugate hyperbolic roots or else two conjugate circular roots; in both cases the roots are planar. What is new in this theory is the treatment of the real roots as conjugate hyperbolic roots. The truth of this principle was made evident by the application to the discharge of a condenser.

Sur la résolution de l'équation du troisième degré. Association française pour l'avancement des Sciences. 1897. In this paper the rules of plane algebra were applied to Cardan's solution of the cubic equation $x^3 + qx - r = 0$; viz.

$$x = \left\{\frac{r}{2} + \sqrt{\frac{q^3}{27} + \frac{r^2}{4}}\right\}^{\frac{1}{3}} + \left\{\frac{r}{2} - \sqrt{\frac{q^3}{27} + \frac{r^2}{4}}\right\}^{\frac{1}{3}}$$

When the quantity under the radical sign is negative, that is in the irreducible case, the binomial expresses a circular complex quantity; and when the quantity under the radical sign is positive, the binomial expresses a hyperbolic complex quantity. The hyperbolic solution has two cases; if $\frac{q^3}{27}$ is negative, the hyperbolic vector belongs to the primary hyperbola; if that quantity is positive, the vector belongs to the conjugate hyperbola. In every case the values of x were deduced by plane algebra, circular or hyperbolic.

Differentiation in space-analysis. Read before the American Mathematical Society in 1895. Science 1: 302. I stated that there were two distinct kinds of differentiation, and that only one of these was treated of in works on quaternions or vector-analysis.

The new more nearly resembled ordinary differentiation, gave a differential coefficient, and allowed Taylor's Theorem to be generalised without difficulty.

Brief of twelve lectures on space-analysis. University of Pennsylvania. University Bulletin, April, 1900. The imaginary expression $i\alpha$ is merely a convenient way of writing $\alpha^{\pi/2}$ and in reality means the same thing. Using this notation, the fundamental principle for quadrants is

$$(i\alpha)\ (i\beta) = -\cos\ \alpha\beta - \sin\ \alpha\beta\ .\ i\ [\alpha\beta].$$

By dividing out ii and the equivalent $-$ we derive

$$\alpha\beta = \cos\alpha\beta + \sin\alpha\beta\ .\ i\ [\alpha\beta]$$

which is the fundamental principle for vectors. This derivation explains the necessary presence of i in the second partial product; a point which is ignored by vector-analysts.

Vector differentiation. Read before the Philosophical Society of Washington. March 31, 1900. Bull. Philos. Soc. Wash. **14**: 73–92. The paper begins by referring to the two kinds of differentiation, depending on the two forms of multiplication. For the direct square

$$d(A^2) = 2AdA$$

and for the direct product

$$d(AB) = AdB + BdA.$$

But for the conjugate square

$$d(AA) = dA\ .\ A + AdA,$$

and for the conjugate product

$$d(AB) = dA\ .\ B + AdB.$$

The latter form is the only kind considered by quaternionists and vector-analysts, and is called differentiation *in situ*.

The application to the modulus and the unit was pointed out. For example, if $R = r\rho$, the direct square is $r^2\rho^2$; and $d(r^2) = 2rdr$, $d(\rho^2) = 2\rho d\rho$. But for the conjugate square $RR = r^2\rho\rho$; and $d(r^2) = 2rdr$ as before, but $d\ (\rho\rho) = d\rho\ .\ \rho + \rho d\rho = 0$. Again $d(\rho^3)$ is simply $3\rho^2 d\rho$; but $d(\rho\rho\rho) = d\rho\ .\ \rho^2 + \rho d\rho\rho + \rho^2 d\rho$.

The paper then proceeds to vector differentiation, by which is meant the theory of ∇. It was pointed out that the differential of a reduced expression is not equivalent to the differential of the primitive expression; and that if the axis ρ is variable, the reduction $\rho^2 = 1$ can be introduced only after the process of differentiation has been completed.

In the investigation of ∇ it is shown that

$$\nabla R = 3;\ \nabla r = 1/\rho;\ \text{and}\ \nabla\rho = 2/r.$$

The next step is to investigate $d(1/\rho)/d\rho$; it is found to be not $-\frac{1}{\rho^2}$ but $\frac{1}{\rho^2}$. The general conclusion was reached that when n is odd

$$\mathrm{d}\,(1/\rho^n)/\mathrm{d}\rho = n\,\frac{1}{\rho^{n+1}};$$

but when n is even, a minus is introduced. The latter statement was afterward found to be erroneous; in no case is there a minus. In this respect the differential of an inverse power of a unit differs from the differential of an inverse power of a modulus.

The rest of the paper was devoted to deriving ∇^2 from ∇ by direct operations of the calculus. For

$$\nabla = \frac{\partial}{\partial x}/h + \frac{\partial}{\partial y}/j + \frac{\partial}{\partial z}/k,$$

it was sought to deduce ∇^2 by means of the multiplication formula

$$\left(\frac{\partial}{\partial x}/h + \frac{\partial}{\partial y}/j + \frac{\partial}{\partial z}/k\right)\left(\frac{\partial}{\partial x}/h + \frac{\partial}{\partial y}/j + \frac{\partial}{\partial z}/k\right);$$

the result was

$$\nabla^2 = \frac{\partial^2}{\partial x^2}/h^2 + \frac{\partial}{\partial y^2}/j^2 + \frac{\partial^2}{\partial z^2}/k^2,$$

which reduces to

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

As the above is the formula for conjugate multiplication, it is evident that what was obtained is the conjugate square of ∇.

It was shown that for a function of r, the spherical modulus,

$$\nabla = \frac{\partial}{\partial r} / \rho \text{ and } \nabla^2 = \frac{\partial^2}{\partial r^2} / \rho^2 + \frac{2}{r}\frac{\partial}{\partial r} / \rho^2.$$

For spherical coördinates,

$$\nabla = \frac{\partial}{\partial r} / \rho + \frac{\partial}{\partial \theta} / r\frac{\partial \rho}{\partial \theta} + \frac{\partial}{\partial \varphi} / r\frac{\partial \rho}{\partial \varphi};$$

and the conjugate square of ∇ was deduced by applying the formula for multiplication, and expanding the resulting terms.

METEOROLOGY.—*On the diurnal variations of atmospheric pressure.* W. J. HUMPHREYS, U. S. Weather Bureau. To appear in full in the Bulletin of the Mount Weather Observatory.

It has been known for nearly two and a half centuries that there are more or less regular daily variations in the height of the barometer, culminating in two maxima and two minima during the course of twenty-four hours; the maxima occurring at 10 o'clock, roughly, forenoon and evening, the minima at 4 o'clock, roughly, afternoon and morning.

Some of the observed facts in regard to this twelve-hour cyclic change of pressure are: (*1*) The amplitude is greatest in the tropics and decreases towards the poles, approximately as the square of the cosine of the latitude; (*2*) The amplitude is everywhere greatest at equinox and everywhere least at solstice; (*3*) The amplitude is greater at perihelion than at aphelion; (*4*) The amplitude is greater by day than at night; (*5*) The amplitude is greatest on clear days and least on cloudy; (*6*) The day amplitude is greater over land than over water; (*7*) The night amplitude is greater over oceans than over continents; (*8*) Over the tropical Pacific Ocean the forenoon barometric maximum is about 1 mm. above and the afternoon minimum 1 mm. below the general average.

Here and there through the voluminous literature on this meteorological mystery one may find an attempt to explain its origin.

The inertia of rising and of falling masses of atmosphere; the formation and evaporation of dew; the pressure due to radiation—"light pressure;" atmospheric tides and a number of other things have been advocated as actual and sufficient causes of the daily variations of the barometer. But a simple numerical test shows that nearly every one of the supposed causes is wholly inadequate to produce the observed pressure changes.

There are however three phenomena which, in the author's opinion, coöperate in such manner as to produce the barometric waves in question.

1. Overflow of the atmosphere from the regions where it is warmest and most expanded towards those where it is coldest and most contracted. The exact hour at which the atmosphere, top to bottom, of any given locality averages warmest and therefore, as a whole, is most expanded depends upon a variety of circumstances, but in general it is not very far from 4 o'clock in the afternoon. Hence at about this time, because of the overflow that the expansion produces, the amount of air overhead, counting from sea level, must be least; and therefore at this same hour a sea-level barometer must have its lowest average reading. Calculations, too long and tedious to include here, based on the average daily change in temperature and on the viscocity of the atmosphere, indicate that the atmospheric overflow resulting from the known thermal changes may fully account for the afternoon or 4 o'clock barometric minimum.

2. Interference by vertical convection with free horizontal flow. It is well known that in general the velocity of the wind increases with increase of elevation, and therefore that the free flow of the atmosphere must be disturbed to some extent by that vertical interchange of its parts which local temperature variations always induce.

We will consider this point a little closer: Let the mass of air m be near the ground and have the horizontal velocity v, and let the larger mass M be at a higher elevation and have, in the same direction, the greater velocity V. If now these two masses of air should mingle in such a manner as to be free from all disturbance, except their own mutual interference, the resulting final

velocity, U, in the same direction, would be given by the equation,

$$U = \frac{mv + MV}{m + M}$$

and there obviously would be no check in the total flow, no damming up and consequent increase of pressure. But this simple mixing of the two masses is by no means the whole story. The rise of the mass m is simultaneously accompanied by the descent of an equal amount from the larger mass M. Thus from a single interchange, due to vertical convection, the total momentum becomes

$$2mv + (M - m)V$$

Hence the total flow is reduced, through ground friction by the amount

$$m(V - v)$$

But as this is for a single interchange, it is obvious that the more active vertical convection becomes, the greater will be its interference with the flow of the atmosphere, the more the winds will be dammed up and the higher the resulting barometric pressure. In general, as convection increases, reaches a maximum and then decreases, so too will the resulting interference go through the same changes.

Now the general movement of the atmosphere is from east to west within the tropics and from west to east at higher latitudes. Hence in either case such damming up of the atmosphere as vertical convection may produce will be essentially along meridians, just as any given phase of vertical convection itself, which has to be substantially at right angles to the march of the sun, is also essentially along a meridian. In other words, convection and its attendant phenomena are functions of the time of day. But, in general, convection increases most rapidly during the forenoon, say 8 to 9 o'clock, is most active at 10 to 11 o'clock, and reaches its greatest elevation about 4 o'clock in the afternoon. Hence the damming up of the atmosphere, due to vertical convection, and the resulting increase of barometric pressure must

increase most rapidly during the forenoon, and come to a maximum about 10 o'clock. After this the convectional interference decreases, while at the same time the amount of atmosphere in a vertical column of fixed cross section also decreases as a result of expansion and overflow, till at about 4 o'clock in the afternoon the barometric pressure, as we have already seen, has reached a minimum.

To form some idea of the magnitude of the barometric change due to convectional turbulence, consider the atmosphere between two parallels of latitude near the equator. This limited quantity of the atmosphere may be regarded as a stream flowing around the earth, having its minimum velocity and maximum depth where convection is greatest, and maximum velocity with minimum depth where convection is absent.

Now the linear velocity of a point on the equator is approximately 28 kilometers per minute, while during the forenoon the rate of increase of the barometric pressure at the same place is roughly 0.2 mm. per hour. Hence a damming up, or check in the flow, of the given stream of atmosphere at the rate of 0.44 kilometer per hour would be sufficient of itself to account for the observed rise in the barometer. But if the average velocity of the wind, or flow of the stream in question, is 10 m. per second, which it may well be, the required change in the velocity could be produced by having, during the course of an hour, only one part in eighty of the whole superincumbent atmosphere in touch with the earth, an amount which, from the size of the cumulus clouds, seems altogether reasonable. Besides, the necessary velocity change is of the same order of magnitude as that observed to take place during, and as the result of, vertical convection.

The afternoon barometric minimum and the forenoon maximum, therefore, are regarded each as an effect of temperature increase; the minimum as due to expansion and consequent overflow; the maximum as mainly caused by vertical convection and consequent interference with the free circulation of the atmosphere.

The forced afternoon minimum would occur in an otherwise stagnant atmosphere, and substantially as at present; but not

so with the forced forenoon maximum since, so far as the interference or damming effect is concerned, it depends upon a flow or circulation of the atmosphere, parallel roughly to the equator.

It remains now to account for the night 10 o'clock maximum and 4 o'clock minimum.

3. Natural or free vibration of the atmosphere as a whole. This subject has been discussed by several mathematical physicists of great eminence. The latest and most complete of these discussions, and the one to which those interested in this phase of the barometric problem are especially referred, is by Lamb,[1] who concludes:

> Without pressing too far conclusions based on the hypothesis of an atmosphere uniform over the earth, and approximately in convective equilibrium, we may, I think, at least assert the existence of a free oscillation of the earth's atmosphere, of "semi-diurnal" type, with a period not very different from, but probably somewhat less than, 12 mean solar hours.

Hence any cause of pressure change, having a semidiurnal period, or harmonic of this, would, if of sufficient magnitude and proper phase, account for the twelve-hour barometric curve. Such a cause, many think, may be found in the irregular daily march of temperature, since the curve expressing this march is more or less approximately resolvable into a diurnal and semidiurnal sine curve. But the resolution is not perfect and besides there is no obvious cause for a temperature increase by night, and hence the reality of the semidiurnal component in the temperature curve is equally doubtful.

All that is needed, apparently, to give the semidiurnal pressure curve is a pressure impulse of the same period, twelve hours, as that of the free vibration of the atmosphere as a whole. And this, it seems, is furnished by the forced forenoon barometric maximum, followed, six hours later, at the same place, by the forced afternoon barometric minumum. In other words, taken together the forenoon and afternoon forced disturbances appear to occur with the proper time interval necessary to set up and maintain the twelve-hour free vibrations of the atmosphere.

[1] Proc. Roy. Soc., A, **84**: 551. 1911.

The course of events at each locality, affecting the height of the barometer, appears to be substantially as follows: (1) A forced forenoon compression of the atmosphere followed by its equally forced afternoon expansion; the two together forming one complete barometric wave, with a 10 o'clock maximum and a 4 o'clock minimum, in harmony with the free vibration of the entire atmospheric shell. (2) Non-disturbance thru the night, or during the period of a single free vibration. (3) Repetition the following day of the forced disturbances in synchronism with, and therefore at such time as to reinforce, the free vibrations.

The series of disturbances of course is indefinitely great, alternately forced and alternately free, but the resulting amplitudes of the barometric changes are limited, thru friction and thru the absence of perfect synchronism, to comparatively small values.

METEOROLOGY.—*Note on the influence of clouds on the distribution of solar radiation.* H. H. KIMBALL and E. R. MILLER. To appear in full in the Bulletin of the Mount Weather Observatory.

The Callendar pyrheliometer as ordinarily exposed gives a continuous record, on a drum revolved by clockwork, of the intensity of the vertical component of the radiation received from both sun and sky. It is a matter of common observation, subconscious in most cases, that with favorable relative position of clouds and sun the solar rays are concentrated. Records obtained by means of the Callendar pyrheliometer at Mount Weather, Va., and Madison, Wis., afford several interesting examples of increased radiation intensity due to such concentration of the sun's rays. A few of these records are reproduced in the more extended note that will appear in the Bulletin of the Mount Weather Observatory. They set forth more clearly than can words the results here given.

At Madison, on February 5, 1912, at 10.40 a.m., a glaringly bright sheet of alto-stratus cloud advanced from the northwest. The recording pen of the register rose as the cloud approached the sun; attained a maximum of 1.11 gram-calories per square centimeter per minute when the edge of the cloud reached the

sun, and then fell rapidly to a value somewhat lower than was recorded before the cloud came up. The record of February 3, 1912, a day without clouds, affords a good comparison curve, as apparently the atmospheric transmission was about the same on these two days, disregarding the effects due to clouds. The value of the sun and sky radiation on the 3d, at the time the peak was recorded on the 5th, was 0.79 calories, so that the radiation of the 5th, augmented by the cloud effect, was 41 per cent in excess of the radiation with the clear sky of the 3d. The cloud was a relatively thin sheet, and the temperature prevailing at the time, 12°F. at the surface, was low enough to make certain that the cloud was made up of ice particles. Its brightness was therefore doubtless due in part to light received upon the upper surface, and transmitted thru the cloud by refraction and reflection.

On July 28, 1912, shortly after 8 a.m., a similar sheet of altocumulus clouds formed rapidly over the Blue Ridge at Mount Weather. An imperfect solar corona was visible soon after the clouds covered the sun. The record made by a horizontally exposed Callendar pyrheliometer shows that as the edge of the cloud sheet approached the sun the concentration of the solar rays caused an increase in the recorded radiation of 13 per cent over what would have been recorded had the clear sky radiation curve been continued. After a short interval of clear sky, clouds again obscured the sun. At this time they were on all sides of it, and just before the sun was obscured the record shows an increase in radiation intensity of 20 per cent above what would have been received had the sky been clear.

At the same time a record was obtained from a Callendar pyrheliometer mounted equatorially in a diaframed tube, and kept pointed towards the sun by clockwork. The angular opening from the center of the pyrheliometric receiving surface to either side of the outer square diafram was about 4°. But little sky radiation could therefore be admitted to the pyrheliometer, and the record shows scarcely a trace of increased radiation intensity as the clouds approached the sun. This seems to disprove the assumption made by some writers that the sky is exceptionally clear between clouds.

On May 17, 1912, a strato-cumulus cloud sheet formed over the Shenandoah valley to the west of Mount Weather, and at 9.15 a.m. advanced rapidly over the mountain. In the five minutes preceding the time at which the sun was obscured the recorded radiation intensity on a horizontal surface increased 12 per cent above the very regular curve that had been made with the clear sky previously prevailing.

At Mount Weather on June 9, 1912, cumulus clouds formed rapidly after 11 a.m., and between 11.03 a.m. and 11.19 a.m., the recorded radiation intensity increased 11 per cent. On July 8, at 10.00 a.m., a thin fracto-cumulus cloud that formed between the zenith and the sun increased the radiation intensity by 8 per cent.

In all these cases the zenith distance of the clouds was less than the zenith distance of the sun, and the clear sky that had preceded the clouds made possible rather definte measurements of the increase in radiation intensity due to the clouds. When these are in rather dense masses, such as cumulus or strato-cumulus formations, the condensation of the solar rays must be attributed to reflection from the cloud surfaces.

There are also many cases in which the cloud effects, while marked, cannot be accurately measured. A good example is the record obtained at Madison on June 3, 1912. The sky was clear until 9.30 a.m., when small cumulus clouds advanced from the northwest and covered the sky by 10.20 a.m. The alternate sunshine and shadow caused the recording pen to traverse the sheet rapidly, for the most part below the normal for that time of the day with a clear sky; but at 11.30 a.m. the relative distribution of clear sky and cloud was such that the unusual concentration of the solar rays caused a radiation intensity of 1.84 calories to be recorded.

Apparently the intensity at this time had the sky been clear would have been 1.12 calories, which gives an excess of radiation of 64 per cent due to reflection from cloud surfaces.

Another notable example of the effect of cloud reflection is the record for June 17, 1912, at Madison, when thin alto-cumulus or alto-stratus clouds prevailed thruout the day. The radiation

intensities fluctuated alternately above and below the highest clear sky records obtained at Madison during June.

By way of contrast, it may be stated that frequently during the prevalence of thunderstorms the dense cloud masses reduce the recorded radiation intensity to less than 1 per cent of clear sky intensities.

Summary. The records from Callendar recording pyrheliometers show that with favorable conditions of sun and clouds the intensity of the vertical component of the radiation received from the sun and sky may be at least 40 per cent in excess of what would have been recorded had the sky been free from clouds, and that an excess of 10 per cent is quite common. In consequence partial cloudiness may diminish but slightly the amount of radiation received at the surface of the earth.

BOTANY.—*Supplement to the Lichen flora of the Santa Cruz Peninsula, California.* ALBERT W. C. T. HERRE.

Some years ago in a paper on lichen distribution in the Santa Cruz peninsula, I expressed the belief that there were probably 200 species and subspecies of lichens in that region. Later in my lichen flora of the Santa Cruz peninsula I described in detail 307 species and subspecies, and diagnosed or mentioned 25 others which I did not deem worthy of very distinct separation. At the same time I stated that this number would be raised materially upon further investigation, and it is my present belief that the lichen flora of the Santa Cruz peninsula comprises not less than 400 good species and subspecies, generally recognized as true lichens. In this estimate no cognizance is taken of those forms (such as the parasitic *Buellias* of authors) usually described in lichen floras, but which in my judgment are true fungi. If in addition one were to include the lichens of the coast about Monterey, and those of the inner Coast Range near San José, including Mt. Hamilton and Alum Rock Park, the number would easily be raised to 500.

The greater number of my as yet undecipherable specimens belong to the genera *Buellia*, *Pannaria*, and *Rhizocarpon*. The

most of these are transition forms which are probably so modified by their habitat as to lose their typical characters and only physiological studies could place them properly.

This paper concludes my studies of the lichens of the Santa Cruz Peninsula as a separate unit. A work on the lichen flora of the Pacific Coast states is in preparation and the receipt of material from any state west of the Rocky Mountain divide would be greatly appreciated. The chief factor to be reckoned with now is that of geographical distribution, and it is therefore important to have as many divergent stations represented as possible. Specimens lent me for examination would be promptly returned, while exchanges would be gladly made and unidentified material named.

VERRUCAZIA NIGRESCENS Pers. in Ust. Ann. Bot. **14**: 36. 1795. We may include here the two following Nylanderian species.

VERRUCARIA AQUILELLA Nyl. *Verrucaria aquilella* Nylander, Flora, **59**: 237, 1876. The variety of *Verrucaria nigrescens* to which Nylander gave this name has been collected by me on sandstone at Devil's Cañon, at an altitude of 2400 feet. It has likewise been collected by Dr. Hasse in southern California. The asci are short-ventricose or saccate and measure 11 to 12μ in breadth by 23 to 24μ in length; when treated with I the thecium is bluish, then yellow or tawny in color; the spores are smaller than in *nigrescen* proper, being from 4.4 to 5.8μ in breadth and 11.7 to 17.5μ in length.

VERRUCARIA FUSCO-CINERASCENS Nyl. *Verrucaria fusco-cinerascens* Nyl. Flora, **59**: 310. 1876. This obscure lichen, said by Leighton to be "apparently only a state or form of *V. nigrescens*," occurs on sandstone along the sea coast near Pigeon Point. The thallus is thin, effuse, of minute dark brown or reddish brown uneven areoles, which may be aggregated into a continuous crust or scattered and nearly disappear. The spores measure 12 by 28μ in our specimens. Not really separable, except in color, from *V. nigrescens*.

VERRUCARIA AETHIOBOLA Wahlenberg. *Verrucaria aethiobola* Wahlb. in Supplem. Ach. Meth. Lich. 17. 1803. Thallus thin, smooth, continuous, effuse, forming dark brown stains on the rocks; no chemical reactions. Apothecia very numerous, hemispherical, rather prominent, more or less immersed, black, minute, 0.3 to 0.1 mm. in diameter; perithecium black, dimidiate; paraphyses wanting; asci cylindrical, pyriform, or variously swollen and irregular, 10 to 14μ broad by 32 to 40μ long; hymenial gelatine blue with I, the spores vinous red, quickly fading to yellow; spores elliptical, 4 to 6.8μ broad by 12 to 15μ long. On rocks in the hills east of Los Gatos, at an elevation of 1500 feet and upward. A European lichen recorded by Dr. Hasse (who has kindly determined this specimen for me) from Santa Monica, California, but apparently not otherwise noticed by American authors.

ARTHOPYRENIA CERASI (Schrad.) Koerber. *Verrucaria cerasi* Schrader Cryptog. Exsicc. 1797. *Arthopyrenia cerasi* Koerber, Syst. Lich. 369. 1855. Thallus very thin, pale yellowish to brown, forming small, determinate, distinct, more or less shining spots which are irregular or more or less rounded in outline, their diameter 8 to 25 mm. Apothecia numerous, often crowded, minute, 0.3 mm. in diameter, convex, shiny black; perithecium black, dimidiate; paraphyses absent; asci tawny with I; spores quadrilocular, 5.75 to 6μ broad, by 20.5 to 27μ long. Rare with us; collected on smooth barked poplars at Stanford University. A fairly common European lichen.

ARTHOPYRENIA SPHAEROIDES (Wallr.) A. Zahlbr. *Arthopyrenia sphaeroides* A. Zahlbr. in Engler and Prantl, Natur. Pflanzenfam. I Teil, Abteil I*, 65. 1903. Thallus thin, uniform, effuse, whitish to greenish olive; darkened by KOH; $CaCl_2O_2$. Apothecia numerous, conspicuous, semi-immersed to sessile, hemispherical to sub-globose, black; ostiolum dot-like or more often not visible; perithecium black, dimidiate; paraphyses hair-like, twining; asci tubular, 8.7μ by 68 to 88μ long; spores 3 to 5.8μ wide by 9 to 13μ long; thecium—reddish brown with I. On *Quercus agrifolia* at Laguna Creek, elevation about 50 feet. Referred here with considerable doubt, rather than to call it new.

ARTHOTHELIUM ANASTOMOSANS (Ach.) Arn. *Arthonia astroidea anastomosans* Ach. Lich. Univ. 146. 1810. *Arthothelium anastomosans* Arnold, Verh. K. K. Zool. Bot. Gesell. Wien, **22**: 304. 1872. Thallus very thin, uniform, gray, forming more or less orbiculate, definite, but small and inconspicuous patches; no chemical reactions evident. Apothecia numerous, minute to small, 0.4 to 0.1 mm. across, black, more or less elevated; of many different shapes, rounded, elongate, and difform, simple, straight, or curved, stellate, and branched; thecium blue with I; asci pyriform or ventricose; spores 6 to 8 in the asci, colorless, with 6 transverse rows of cells and 2 longitudinal rows, 6 to 8.8μ broad by 14.6 to 19μ long. On twigs of *Cupressus macrocarpa* and *Pinus radiata* at Stanford University. By the spores this is *anastomosans*, but it does not agree very well in other respects. Apparently rare, but perhaps really passed over as *Arthonia radiata*; a microscopic examination is the only sure test. Recorded also by Dr. Hasse from Catalina Island. A European lichen of widespread distribution.

LECIDEA ATROBRUNNEA (Ram.) Schaerer. *Lichen atro-brunneus* Ramond in DC. Fl. Fr. **2**: 367. 1803. *Lecidea atro-brunnea* Schaerer, Spicilegia, 134. 1828. Tuckerman, Synopsis, **2**:74. 1888. Thallus of thick areoles or squamules which may be thinly scattered, closely set, or crowded, upon a conspicuous black limiting hypothallus; areoles from small and flat soon swollen and convex, with a thin black margin or abnormally with pale or white edges; circular or irregular in shape, finally with wavy and sculptured surface, the marginal areoles sometimes elongate and lobate; color brown, varying from pale and yellowish to copper, chestnut, and blackish brown, smooth and shiny to dull; medulla blue or violet with I. Apothecia small to medium, rarely large, 0.5 to 1.8 mm. in diameter; closely adnate, flat or slightly hollowed, black tho sometimes pruinose, with an entire, thick, elevated, concolorous margin; finally more or less flexuous with wavy or crenate margin, and the disk more or less swollen, and thru crowding or fusion forming irregular clusters, when the apothecia seem to be very large; hypothecium pale to brownish; epithecium thick, greenish black; paraphyses slender, straight, their slightly enlarged tips and upper portion greenish; thecium

blue with I; asci short, narrowly clavate, in our material usually without well developt spores; these often smaller than given by most authors, 2.8 to 3.5μ wide and 5.8 to 8μ long; in a specimen collected by Bolander in the Yosemite Valley and determined by Tuckerman I find the pruinose apothecia have spores 3.5 to 5μ wide by 9 to 11μ long. A lichen of granitic and schistose rocks in alpine and arctic regions. Occurring on Black Mountain, at an altitude of 900 meters. Common in the Sierra Nevada at 2000 meters and above and one of the most successful of lichens on the highest peaks. On Mt. Hamilton, near San José, at 1400 meters. Collected in southern California by Dr. Hasse at 1800 meters and above. Abundant in western Nevada. Recorded by Tuckerman from a number of Californian localities, Mt. Hood, Oregon, the Oregon coast, and from Washington. Found on all high mountains thruout the region west of the Missouri River.

LECIDEA SILACEA Ach. *Lecidea silacea* Ach. Meth. Lich. 48. 1803. *Lecidea lapicida* Tuck. Synopsis, **2**: 70. 1888. in part. Thallus indeterminate, thin or scanty, the areoles rather thickish, more or less tartareous, ashen gray in our specimens but also glaucous; KOH −; $CaOCl_2$ faintly reddish; KOH + $CaOCl_2$, red; medulla faint blue with I. Apothecia 0.3 to 1.3 mm. in diameter, closely adnate or sometimes somewhat immersed, numerous, often clustered; from circular becoming irregular by crowding, the flat black disc becoming slightly convex; margin thin, black, elevated, said to finally disappear tho not so in our material; the broad hypothecium black or very dark brown; epithecium greenish umber; thecium more or less greenish, becoming deep blue with I; paraphyses conglutinate, not very distinct, with enlarged darker tips; asci clavate; spores short ellipsoid, 4 to 6μ broad, and 9 to 12μ long. On sandstone in the Santa Cruz Mountains at from 600 meters (Bear Gulch road) to 1000 meters elevation (Castle Rock). Occurring also at Mt. Hamilton at 1400 meters. Undoubtedly the "Glaucous lichen" reported by Tuckerman from the mountains of California. A European lichen occurring also in various parts of North America and in New Zealand.

LECIDEA GLEBULOSA (E. Fries). *Biatora glebulosa* E. Fries, Lich. Europ. Reform. 252. 1831. Tuckerman, Synopsis, **2**: 16. 1888. Thallus of small, closely appressed scales which vary from crumb-like to lobed and crenate; dispersed, or crowded and forming an irregular uneven crust which is effigurate at the margin when well developt; dull whitish to gray in color. KOH −; $CaCl_2O_2$, red. Apothecia appressed, large, the flat or slightly convex disk usually black, but varying from reddish or pale flesh color to black; the thick, prominent, paler or whitish margin becoming irregular and finally deeply folded so that the disk is more or less dissected; hypothecium pale brown; spores broadly ellipsoid 5 to 6μ broad and 10 to 12μ long. On sandstone in the foothills near Stanford University, but rarely fertile. In the Oakland Hills it is not rare and is abundantly fertile. Recorded by Tuckerman from California, Oregon, and Washington.

LECIDEA ULIGINOSA (Schrad) Ach. *Lichen uliginosus* Schrader, Spic. Fl. Germ. 88. 1794. *Lecidea uliginosus* Ach. Meth. Lich. 43. 1803. *Biatora uliginosa* Tuckerman, Synopsis, **2**: 27. 1888. Thallus spreading indefinitely, of minute to microscopic granules which form a very thin and more or less continuous blackish, brown, or dusky crust; no reactions with the usual reagents. Apothecia of the same color as the thallus, small to very minute, 0.2 to 0.6 mm. in diameter, at first immersed but mostly adnate; with flat disk which is rarely convex; margin thin, elevated, at first pale but soon blackening and disappearing; hypo-

thecium dark blackish brown to brown; epithecium dark reddish brown; thecium brown, the paraphyses not distinct, coherent, simple, blue with I; asci cylindrical or narrowly clavate; spores ovoid or broadly ellipsoid, 4.9 to 7.5μ broad and 9.75 to 14.7μ long. On damp earth and moist shaly rock in the city of Santa Cruz, at an elevation of about 50 feet. Recorded by Tuckerman from Washington and from British Columbia by Macoun. Reported from most parts of North America and Europe, and also from Asia.

CATILLARIA LAURERI Hepp. *Catillaria laueri* Hepp in Arn. Exsic. No. 353. 1867. *Biatora laureri* Tuck. Synopsis, 2: 30. 1888. Thallus very thin and scanty, or even disappearing, or at times roughened and contiguous to chinky, scurfy or smooth; effuse, forming small, irregular dusky ashen to ashy white patches; turning brown with KOH. Apothecia rather numerous, minute to small (0.4 to 1 mm. in diameter), the black, slightly roughened disk at first plane, but soon convex and swollen, when the thin entire margin disappears and the fruit is lecideine; epithecium thick, granulose, blackish or black violet; thecium pale or colorless; with a violaceous-reddish cast, becoming deep blue with I; hypothecium dusky, dusky reddish, and nearly clear, but when seen in thick sections it is of a dull greenish color; paraphyses thread-like, more or less capitate and dusky tipped, free, somewhat lax and twining; asci clavate, blue with I; spores bilocular, fusiform-ellipsoid, 3 to 5μ broad and 8.7 to 16μ long. Collected but once by me, on dead twigs along the sea coast near Pigeon Point. A bark lichen of Northern and Central Europe, reported by Tuckerman from New England, New York, California, Oregon, Alaska, and Canada.

THELOCARPON ALBO-MARGINATUM Herre, new species. Thallus composed of very small to minute (0.1 to 0.3 mm. in diameter), more or less circular and rather thick but flattened thalline warts, which lie rather thinly scattered over the substratum, or occasionally are grouped into small clusters; color a dark chestnut brown, with a distinct white margin surrounding each areole. One apothecium in each areole, its presence indicated by a minute pore; asci flask shaped; paraphyses free, thread-like, and more or less twining, not septate and rarely forked near the apex; thecium blue with I; spores very numerous, ellipsoid, 1 to 1.5μ broad, 2.5 to 4μ long. On a loose rock lying on earth near the summit of the west wall of Devil's Cañon, elevation about 2400 feet. Externally the plant much resembles a degenerate *Acarospora* and is very likely to be passed over as such.

ZAHLBRUCKNERELLA Herre. *Zahlbrucknera* Herre, Proc. Wash. Acad. Sci. 12: 129. 1910, not Reicheub. 1832. As the name of *Zahlbrucknera* had already been applied to a genus of flowering plants, my use of the name was untenable; I therefore make the modification given above.

COLLEMA CRISPUM (Huds.) Hoffm. *Lichen crispus* Hudson, Fl. Ang. 447. 1762. *Collema crispum* Hoffm. Deutsch. Fl., 2: 101. 1795. Thallus small, effuse, forming close or scattered greenish black or brownish black clumps, 6 to 20 mm. in diameter; the minute lobes outspread and flattish, or usually more or less erect and crowded, with crisped and intricate or dentate-granulate margins. Apothecia small to medium, 0.5 to 2 mm. in diameter, with concave reddish-black disk, which at last may become plane; margin thick, sub-entire or granulate; spores oblong ellipsoid, four-locular, colorless or faintly brown, measuring 7.3 to 11.7μ in breadth by 17.5 to 29μ in length; said to become more or less muriform, tho not so in our scanty specimens. Rare; on sandstone at Castle Rock, altitude

3000 feet. A European earth-dwelling lichen which is found across the northern half of the American continent.

LEPTOGIUM CRISTATELLUM (Tuck.) Herre. *Collema cristatellum* Tuck. Lich. Calif. 29. 1866. Herre, Proc. Wash. Acad. Sci. 7: 378. 1908. A careful study of the thallus of this lichen shows it to be a true *Leptogium* and not a *Collema*, a well developed cortical layer being present. In addition to the stations previously recorded, I have found it growing on crumbling sandstone beside the road en route to the Lick Observatory in the inner Coast Range, at an altitude of about 1100 feet.

PLACYNTHIUM SONOMENSIS (Tuck.) Herre. *Pannaria sonomensis* Tuck., Proc. Am. Acad. 12: 169 1877; Synopsis, 1: 126. 1882. Thallus forming small to very small orbiculate or irregular dark dull brown patches, which appear to the naked eye as little more than dirty stains on the rocks; made up of very minute but distinct elongated, terete, linear and many cleft lobes; these more or less tangled and coralloid centrally, but somewhat expanded marginally; beneath white or pale, and smooth; upon a black hypothallus, which is usually obsolete; alga *Scytonema*. Apothecia minute, lecanorine, sessile; the flat disk soon convex and finally sub-globose, in which case the persistent entire whitish margin may be excluded; color reddish brown and blackening; margin enclosing algae which likewise form a layer beneath the colorless to very pale dusky hypothecium; epithecium broad, umber; thecium colorless, blue with I; paraphyses, thick, jointed, with enlarged tips; asci much shorter than the paraphyses, clavate, their contents usually not differentiated, 6 to 9μ broad by 35 to 41μ long; spores very slender, straight or slightly curved, simple, colorless, 1.5 to 3μ in breadth and 17 to 26μ in length; these measurements are somewhat smaller than those given by Tuckerman. I have collected only sterile specimens in the Santa Cruz Peninsula, at Castle Rock, altitude 3000 feet; on rocks at Alum Rock Park, inner Coast Range, near San José, at an altitude of 500 feet, it occurs abundantly fertile. Collected on various rocks in Sonoma County and also at Yosemite by Bolander. This plant does not agree with any of the genera as defined by Zahlbruckner, as it combines lecanorine apothecia, simple spores, and *Scytonema* algae. These characteristics do not occur together in any genus as defined, but as the separation of the genera of the *Pannariaceae* should be based primarily upon the alga enslaved rather than upon the spore characters or apothecial structure, I deem it best to call this plant a *Placynthium*.

PANNARIA LEUCOSTICTA (Tuck.) *Pannaria leucosticta* Tuckerman, Proc. Amer. Acad. 4: 404. 1860. Cummings, Williams, and Seymour, No. 270, Decades. North Am. Lichens, Sligo Creek, Maryland. Thallus small to medium, appressed, the radiate lobes expanded marginally, their tips denticulate crenate, upturned; from sub-orbicular becoming effuse; centrally becoming reduced to minute denticulate squamules and passing into a granular crust; color whitish gray, changing centrally to brownish or greenish buff; margin of lobes and finally the whole upper surface except the peripheral lobes covered with minute, erect, denticulate, whitish, bluish, or concolorous granules; beneath white, and blackening; no chemical reactions. Apothecia numerous, of medium size; appressed, with dentate thalline margin, the flat disk soon strongly convex; color pale to dark red-brown; epithecium pale yellow; hypothecium brownish; thecium pale blue, then more or less tawny with I; spores simple, subglobose to broadly ellipsoid, often pointed at one

end, 9.5 to 10μ broad by 10 to 19.5μ long. Encrusting bark, mosses, and *Sticta anthraspis*, on the trunk of a scrub oak in the chaparral east of Los Gatos, at an altitude of about 2000 feet. Common thruout eastern North America and reported by Macoun from British Columbia, but not otherwise recorded west of the Rocky Mountains.

LECANORA BADIA (Pers.) Ach. *Lichenbadius* Persoon, Ust. Ann. Bot. 7: 27. 1794. *Lecanora badia* Ach. Lich. Univ. 407. 1810. Tuck. Synopsis N. Am. Lich. 1: 190. 1882. Thallus small, indeterminate, on a thin black hypothallus; of fissured areoles which pass into squamules with imbricate-areolate surface; color olive brown, usually more or less polished, but rather dull in our specimens; no reactions to chemicals. Apothecia small to minute, sessile; flat in our specimens tho said to become convex; disk blackish brown and reddish-black, with a thick, entire, persistent margin which is concolorous with the thallus, or more often blackens; paraphyses stout, their tips brownish; thecium bluish with I; spores broadly spindle shaped, 3 by 9μ; according to Tuckerman they measure 3 to 5μ in breadth by 10 to 14μ in length. A few obscure but well marked specimens were collected on rocks at Twin Peaks, San Francisco, at an elevation of about 700 feet. Recorded by Tuckerman from the White Mountains, Tadousac, Canada, and Arctic America. Beyond doubt occurring in the northern part of the Sierra Nevada and in the Cascade Mountains. Common enough in northern and central Europe in both alpine and maritime situations.

ABSTRACTS

Authors of scientific papers are requested to see that abstracts, preferably prepared and signed by themselves, are forwarded promptly to the editors. Each of the scientific bureaus in Washington has a representative authorized to forward such material to this journal and abstracts of officia' publications should be transmitted through the representative of the bureau in which they originate. The abstracts should conform in length and general style to those appearing in this issue.

PHYSICS.—*Circular dichroism and rotary dispersion of certain salt solutions.* L. B. OLMSTEAD. Physical Review, **35**: 31–46. 1912.

The author investigated the rotation and ellipticity produced in plane polarised light by transmission thru solutions containing an optically active acid radical combined with an absorbing metallic ion; and, in connection with this, the extinction coefficients and refractive indices of the same solutions. Readings were made at intervals of $10\mu\mu$ to $30\mu\mu$ thruout the visible spectrum and the values obtained were plotted against wave-lengths. The colored compounds successfully used to give circular dichroism were chromium, copper, cobalt, and manganese; and the organic compounds were tartrates, malates, and lactates. The ellipticity is of the order of 10^{-3} to 10^{-2} for the concentrations used and is always a maximum in the region of maximum absorption. The rotation is anomalous in every case but the refractive index shows no anomaly. In the chromium compounds the absorption curves are of the same general shape with an absorption band in the yellow and one in the violet. At about $570\mu\mu$ the slope of the rotation curve is steepest and usually changes sign; and at this point the ellipticity reaches a maximum positive or negative value. In the copper compounds the ellipticity and absorption are maximum in the red, decreasing rapidly toward the blue. The cobalt compounds reach their maximum ellipticity and absorption in the green. The manganese tartrate solution shows an absorption band in the violet and a slight one in the green. The ellipticity changes rapidly from a negative value at $420\mu\mu$ to a maximum positive value at $540\mu\mu$, decreasing again to a small negative value in the red. L. B. O.

PHYSICS.—*The relation of the horsepower to the kilowatt.* Circular 34, Bureau of Standards, 1912.

A unit of power should not vary in magnitude from place to place, and hence the horsepower should be defined in such a way as to be equivalent to a fixed number of watts. The Standards Committee of the American Institute of Electrical Engineers in 1911 adopted the most frequently used equivalent, namely, 746 watts, as the precise value of the horsepower and this value is now recommended by the Bureau of Standards. The "continental horsepower," used in Europe, is similarly best defined as 736 watts. In accord with the trend of modern practice, particularly in electrical engineering, it is recommended that the kilowatt instead of the horsepower be used generally as the unit of power.

J. H. DELLINGER

ELECTRICITY.—*Electric wire and cable terminology.* Circular 37, Bureau of Standards, 1912.

On account of the growing need for precise definitions in the field of conductor terminology, the Standards Committee of the American Institute of Electrical Engineers requested the Bureau of Standards to prepare a circular on the subject. As a result of extensive correspondence and consultation it was found possible to formulate a reasonably consistent body of definitions without introducing radical departures from existing general practice. Seventeen of the most important terms have been defined.

It was found necessary to give the most care and thought to the terms "strand" and "cable." In the strong preponderence of current opinion, "strand" implies a component part of a cable or stranded conductor, each part being either a combination of wires or a single wire. It fortunately happens that this is precisely in accord with the non-technical meaning of "strand." A "cable" is defined as either: (1) a stranded conductor, i.e., a conductor composed of a group of wires; or (2) a combination of conductors insulated from one another. The first kind of cable may be either bare or insulated. The component conductors of the second kind of cable may be either solid or stranded and the whole may or may not have a common insulating covering.

From the main terms defined, the minor germs should follow logically and most or all of the confusion which has existed in this subject should disappear.

J. H. DELLINGER.

ELECTRICITY.—*The testing of instrument transformers.* P. G. AGNEW and F. B. SILSBEE. Proceedings American Institute Electrical Engineers. June, p. 1267. 1912.

This paper describes a new arrangement for testing instrument transformers, which has recently been developed at the Bureau of Standards. For current transformers the ratio is found as formerly by balancing the IR drops in two resistances, but the phase angle is measured by using the secondary voltage of a mutual inductance to balance the quadrature component of the voltage drops. A vibration galvanometer is used as a detector, and when a complete balance is obtained both ratio and phase angle can be computed from the values of the resistances and the mutual inductance.

For potential transformers a high resistance is connected across the primary side and the secondary voltage is applied through the galvanometer potentiometer fashion to a portion of this resistance. The phase relations are taken care of by inserting a large fixed self inductance in the low side of the resistance and then shunting a fixed condenser around a variable portion of the latter. This has the effect of neutralizing a portion of the inductance.

The vibration galvanometer used was of the Campbell type with a moving coil specially constructed to give high voltage sensitivity. The instrument could detect 0.5 microvolt at 25 cycles.

The advantages of this method are that only a single instrument is required for all ranges of transformers, but one observer is required, and neither a polyphase source, a phase shifting device nor a rotating commutator is required. P. G. A.

GEOLOGY.—*Coal near the Black Hills, Wyoming-South Dakota.* R. W. STONE. Bulletin U. S. Geological Survey No. 499, pp. 66, with maps, sections, and illustrations. 1912.

The salient features of the Black Hills uplift are: (1) a central area of crystalline Archean and Algonkian rocks; (2) a limestone plateau with in-facing escarpment; (3) a continuous trough, the Red Valley, completely encircling the plateau; and (4) an outer rim of flat-topped ridges sloping away from the central area. The formations of which it is necessary to take cognizance in connection with the occurrence of coal are the Spearfish formation (Triassic?), locally known as the "red beds;" the Sundance formation and Morrison shale (Jurassic?); and the Lakota sandstone, Fuson shale, Dakota sandstone, and Graneros shale (Cretaceous).

Coal is found on the west and south sides of the uplift at or near the base of the Lakota sandstone. All known occurrences are described and the conclusion is reached that Cambria is the only locality which will add to the record of coal produced in the Black Hills.

An interesting feature of the Cambria coal is the occurrence in it of gold, assays showing up to $2 a ton. Four kinds of coal are described: bituminous, cannel, splint, and "pine needle," all occurring in the same bed. The latter kind, resembling a mass of pine needles, is composed of fibers of resin, circular to elliptical in cross section, which may originally have filled some cylindrical vegetable cell. R. W. S.

BOTANY.—*Observations sur les quelques espèces indochinoises des genres Atalantia et Glycosmis.* WALTER T. SWINGLE. Notulæ systematicæ, H. Lecomte, **2**: 158–163, fig. 1^{1-6}, fasc. 5, 20 Dec. 1911, (p. 158–160) and fasc. 6, 25 Mar. 1912 (p. 161–163).

A study of the material from Indo-China in the Herbarium of the Muséum d'Histoire naturelle, Paris showed that the plant recently listed by M. A. Guillaumin in the Flore générale de l'Indo-Chine as *Atalantia disticha* (Blanco) Merrill, constitutes in reality a new species, *A. Guillaumini*, Swingle. The former plant, common in the Philippine Islands, is not yet known from the Asiatic mainland.

This new species differs decidedly from *A. disticha* in having large (nearly 1 inch in diameter) pulpless fruits containing very large seeds. It also has larger leaves with darker colored, less pubescent petioles. *A. Guillaumini* resembles *A. ceylonica* in having large seeds which almost completely fill the fruit, but the veins are not curved as in this latter species, nor are the stipules so large and foliaceous. This new species should be tested as a stock for *Citrus*.

A close study of the type material of *A. pseudoracemosa*, Guill., showed striking concordance with the characters of the genus *Glycosmis*, especially in the fine ferrugineous pubescence of the young leaves. Until the plants included in the genus *Glycosmis* are better known, the species in question should be called *Glycosmis pseudoracemosa* (Guill.) Swingle.

Glycosmis seems to include three types of species: (1), with compound leaves like *G. sapindoides*; (2), with simple leaves but long, slender petioles articulate at both ends, like *G. Bonii*; (3), with simple leaves and short petioles not articulate with the blade, like *G. pseudoracemosa*. These three groups are more or less united by the type species of the genus, *G. cochinchinensis*, which has polymorphous leaves.

MAUDE KELLERMAN.

BOTANY.—*New or noteworthy plants from Colombia and Central America* —3. H. PITTIER, in Contributions from the U. S. National Herbarium, vol. 13, pt. 12, pp. 42, text fig. 32 and plates 19. 1912.

This is the third of a series of papers in which the author describes new materials collected during his travels, occasionally entering into critical examination of old genera and species or revising and complementing the descriptions of species that, altho long known, have more lately acquired some importance in economic botany. A thoro taxonomic investigation of the tropical American tribe of the Olmediae (Moraceae) brought to light the singular mistake made by suppressing the well-founded genus Naucleopsis, partly on account of a fantastic plate in the *Flora brasiliensis* and partly through the misplacing of an illustration in the *Pflanzenfamilien*.

With reference to the transfer of *Moquilea platypus* Hemsl. to the genus Licania, it should be stated that this had already been made by Dr. Fritsch in 1889 and therefore the new combination should read *Licania platypus* (Hemsl.) Fritsch. This is a tree that bears heavy crops of a fruit of excellent quality and it may, on that account, become of some economic importance. The same applies also to *Mammea americana* and several other Guttiferae; some of which are new and belong to the genus Rheedia. Of the Sapotaceae several new species are described, and the descriptions and synonymy of old ones revised and somewhat modified. H. P.

REFERENCES

METEOROLOGY.—Recent publications by the U. S. Weather Bureau: *Monthly Weather Review*, **39**: No. 12, pp. 1791–1949, charts 8, 1911; **40**: No. 1, pp. 1–161, charts 8, 1912; **40**: No. 2, pp. 163–320, charts 8, 1912; **40**: No. 3, pp. 321–486, charts 8, 1912; **40**: No. 4, pp. 487–658, charts 8, 1912; **40**: No. 5, pp. 659–813, charts 8, 1912.

The relation between the precipitation over the watershed of the Ohio River above and the stream-flow at Cincinnati. J. WARREN SMITH. **40**: pp. 40, charts 4, 1912.

Forecasting frost in the North Pacific States. EDWARD A. BEALES. **41**: pp. 49, charts 6, 1912.

Summaries of climatological data by sections. Bulletin W: vol. i. *Sections west of the Mississippi River*, vol. ii. *Sections east of the Mississippi River.* The numerous charts and 600 pages, about, of each volume give, chiefly in tabular form, a vast amount of climatological data.

HYDROLOGY.—*Gazetteer of surface waters of California:* Part 1. Sacramento River Basin. Prepared under the direction of John C. Hoyt, by B. D. Wood. Water-Supply Paper U. S. Geological Survey No. 295. Pp. 99. 1912.

Surface water supply of the United States, 1910: Part II. *South Atlantic Coast and Eastern Gulf of Mexico.* H. R. MALL and J. G. MATHERS. Water-Supply Paper U. S. Geological Survey No. 282. Pp. 109, views and sections. 1912. Part V. *Hudson Bay and Upper Mississippi River.* ROBERT FOLLANSBEE, A. H. HORTON, and G. C. STEVENS. Water-Supply Paper U. S. Geological Survey No. 285. Pp. 318, views and sections. 1912. Part VI. *Missouri River Basin.* W. A. LAMB, W. B. FREEMAN, RAYMOND RICHARDS, and R. C. RICE. Water-Supply Paper U. S. Geological Survey No. 286. Pp. 308, views and sections. 1911. Part VII. *Lower Mississippi Basin.* W. B. FREEMAN and J. G. MATHERS. Water-Supply Paper U. S. Geological Survey No. 287. Pp. 91, views and sections. 1911.

Water Resources of the Penobscot River Basin, Maine. H. K. BARROWS and C. C. BABB. Prepared in cooperation with the Maine State Survey Commission. Water-Supply Paper U. S. Geological Survey No. 279. Pp. 285, maps, diagrams, and sections. 1912.

Gaging stations maintained by the United States Geological Survey, 1888–1910, and Survey publications relating to water resources. B. D. WOOD. Water-Supply Paper U. S. Geological Survey No. 280. Pp. 102. 1912.

ENGINEERING.—*Results of triangulation and primary traverse for the years 1909 and 1910.* R. B. MARSHALL. Bulletin U. S. Geological Survey No. 496. Pp. 392, with map. 1912.

PROCEEDINGS OF THE ACADEMY AND AFFILIATED SOCIETIES

GEOLOGICAL SOCIETY OF WASHINGTON

The 257th meeting was held in the Cosmos Club April 24, 1912, Vice-President White in the chair. The following informal communications were presented:

Relation of the Sewanee conglomerate to the Bon Air sandstone: CHARLES BUTTS.

A zircon rock from Virginia: F. L. HESS.

Koenigsberger on geothermic gradients and petroleum: E. W. SHAW. Mr. Shaw reviewed a paper by Professor Koenigsberger which treats of the technique of measuring underground temperatures, of the temperature gradients in numerous borings, mines and tunnels for which data are given, and of the conclusions that may be drawn therefrom. In the computation of the gradients in mines and tunnels the observed temperatures are corrected for the effect of forced ventilation, water evaporation and movement, the shape of the surface and other factors which must be considered in working out the isogeotherms. Professor Koenigsberger also takes into account the difference in heat conductivity of different rocks, or of the same rocks in different directions with reference to the cleavage, and the effect of water in the rocks of nearby large bodies of water on the surface. He considers the age of the strata, their attitude, their degree of metamorphism, the presence or absence of evidence of vulcanism, the heat generated by the oxidation of iron sulfide and hydrocarbons, the hydration of anhydrite and other minerals, the depolymerization of bituminous substances, especially coal and oil, and, finally, radioactivity. After evaluating all these factors, he arrives at the conclusion that whereas the average rate of increase in temperature within the earth is about 1°C. for 30 m., in coal regions there is a higher gradient, in oil and gas fields still higher, and that this fact can be used in prospecting.

Professor Koenigsberger's observations seem to have been made with care and precaution, but there appear to be weak points. (1) The data from mines and tunnels are subject to so much correction for circulating air and water, and the arrangement of mountains and valleys at the surface, etc., that one hesitates to put much weight on them, especially as slight error would make a great difference in the conclusion. (2) Of the fifty or sixty measurements in excavations of all kinds, only a few were made by the author himself and none was made with instruments more accurate than the best maximum mercury thermometer, which is

only accurate to about 1°. (3) The given temperature gradients in oil fields are not uniformly high, but are of all degrees of steepness, and it is only true that of the gradients given the *average* for oil wells is higher than that for water wells. Perhaps if the gradients for fifty other wells were considered the opposite conclusion might be drawn. (4) The temperature of wells in the Appalachian and interior oil fields certainly does not increase nearly so rapidly as would be necessary according to Professor Koenigsberger for the existence of considerable pools of oil and gas. Measurements taken in western Pennsylvania indicate a rise of only about 1° for 75 to 100 feet. It is possible that chemical changes in oil and coal, especially those which lead to more stable compounds, may generate heat, but whether enough heat could be generated to warm up the outer mile or two of the earth perceptibly is perhaps open to question. Altho it may be to some degree true one feels a hesitancy about accepting without further confirmation the conclusion that the presence or absence of oil and other valuable minerals can be foretold by the temperature gradients within the earth.

REGULAR PROGRAM

Glacial deposits in Roxbury conglomerate: LAURENCE LA FORGE.
Soil movements in Alaska: B. L. JOHNSON.
Origin of freshwater lime deposits: CHARLES A. DAVIS.